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Team No. : ____________ Date : ____________ Lecturer's Signature : ____________

Introduction

Goals

- Measure the inductive time constant (τ_{RL}).
- Understand the inductive reactance (X_L).
- Understand the phase shift between current and potential difference.

Theoretical Backgrounds

1. **Potential Difference**
   (a) Faraday's law of induction states that the emf of a circuit is generated by
   $$\mathscr{E} = -\frac{d\Phi_B}{dt},$$
   if the magnetic field flux,
   $$\Phi_B = \int \boldsymbol{B} \cdot d\boldsymbol{A},$$
   varies with time.
   (b) The inductance L is constant if the geometric structure of an inductor is stationary. In that case, $L = \Phi_B / I$.
   (c) The potential difference v_L between the two terminals of an inductor of inductance L is
   $$V_L = L\frac{di(t)}{dt}.$$
2. **Inductance**
   Consider a solenoid of length ℓ, cross sectional area A, and the number of turns N.
   (a) Ampere's law can be used to compute the flux passing through the solenoid as
   $$\Phi_B = \mu_0 \frac{NIA}{\ell},$$
   where I is the current and μ_0 is the permeability of vacuum.
   (b) The inductance is given by
   $$L = \mu_0 \frac{N^2 A}{\ell}.$$
   (c) It is straightforward to derive the formula for the equivalent inductance for a serial connection of N inductors L_i's:
   $$L = \sum_{i=1}^{N} L_i.$$
   (d) The equivalent inductance for a parallel connection of N inductors L_i's is:
   $$\frac{1}{L} = \sum_{i=1}^{N} \frac{1}{L_i}.$$
3. **Kirchhoff's Rule**
   The algebraic sum of the changes in potential during a travel over a closed loop vanishes. The formula is
   $$\sum_{i=1}^{N} \Delta V_i = 0,$$
   where i indicates each circuit element on that closed path.
4. ***RL* circuit with a square wave input**
   (a) The potential equation is
   $$v_L + v_R = \mathscr{E},$$
   where $\mathscr{E}$ is the potential difference between the two terminals of the battery.
   (b) This is the first-order linear differential equation:
   $$L\frac{di}{dt} + Ri = \mathscr{E}.$$
   (c) If $I(t=0) = 0$, then
   $$\begin{aligned} v_L(t) &= L\frac{di}{dt} = \mathscr{E} e^{-t/\tau_{\mathrm{RL}}}, \\ v_R(t) &= Ri = \mathscr{E}(1 - e^{-t/\tau_{\mathrm{RL}}}), \end{aligned}$$
   where τ_{RL} is the ***time constant***,
   $$\tau_{\mathrm{RL}} = \frac{L}{R}.$$

(d) If $I(0) = I_0$ $[\frac{dI}{dt}(t=0) = 0]$ and we set $\mathscr{E} = 0$ at $t = 0$, then

$$\begin{aligned} v_L(t) &= L\frac{dI}{dt} = -RI_0 e^{-t/\tau_{\mathrm{RL}}}, \\ v_R(t) &= RI = RI_0 e^{-t/\tau_{\mathrm{RL}}}. \end{aligned}$$

5. ***RL* circuit with a sine wave input**

(a) This is the first-order linear differential equation with a sine wave input:

$$v_R + v_L = Ri + L\frac{di}{dt} = \mathscr{E}_m \sin\omega_{\mathrm{d}} t,$$

where ω_{d} ($\mathscr{E}_m$) is the frequency (amplitude) of the sine wave input.

(b) The current $i(t)$ of the circuit is

$$i(t) = \frac{\mathscr{E}_m}{Z}\sin(\omega_{\mathrm{d}} t - \phi),$$

where Z is

$$Z = \sqrt{R^2 + X_L^2},$$

X_L is the ***inductive reactance***

$$X_L = \omega_{\mathrm{d}} L,$$

and ϕ is the ***phase constant***

$$\phi = \arctan\frac{X_L}{R}.$$

(c) The potential difference $v_R(t)$ of R is

$$\begin{aligned} v_R(t) &= i(t)R \\ &= \frac{\mathscr{E}_m R}{Z}\sin(\omega_{\mathrm{d}} t - \phi). \end{aligned}$$

The phase of $i(t)$ and that of $v_R(t)$ are **in phase**.

(d) The potential difference $v_L(t)$ of L is

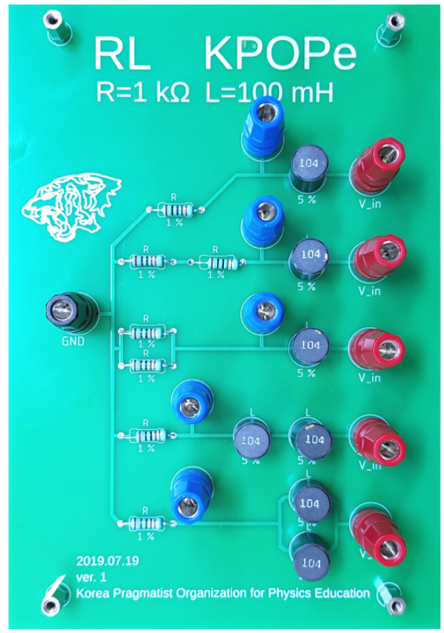
$$\begin{aligned} v_L(t) &= L\frac{di(t)}{dt} \\ &= \frac{L\omega_{\mathrm{d}}\mathscr{E}_m}{Z}\cos(\omega_{\mathrm{d}} t - \phi) \\ &= \frac{L\omega_{\mathrm{d}}\mathscr{E}_m}{Z}\sin(\omega_{\mathrm{d}} t - \phi + \frac{\pi}{2}). \end{aligned}$$

The phase of $v_L(t)$ **leads** that of $i(t)$ by $\frac{\pi}{2}$, or the phase of $i(t)$ **lags** that of $v_L(t)$ by $\frac{\pi}{2}$.

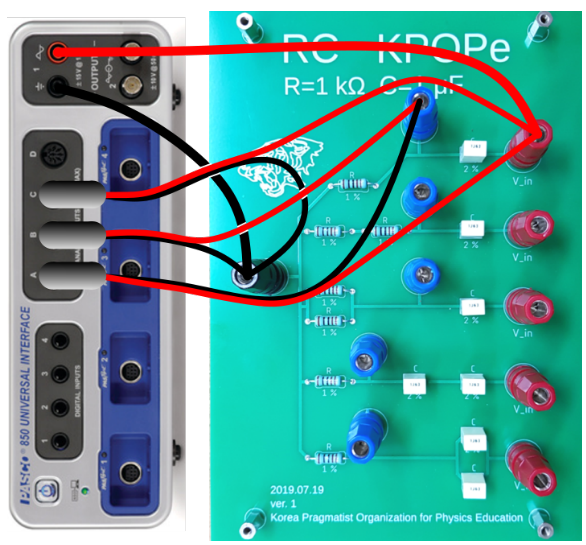
(e) The phase of $v_L(t)$ **leads** that of $v_R(t)$ by $\frac{\pi}{2}$, or the phase of $v_R(t)$ **lags** that of $v_L(t)$ by $\frac{\pi}{2}$.

Instrumentation

1. **Board *RL***



2. **Connection**



Experimental Procedure

1. **E1: a square wave input**
   (a) Carry out experiment with the five circuits on the board RL.
   (b) Connect the signal generator to V_{in} and the ground of the board.
   (c) Connect the voltage sensor across the ground and V_{in}. Also connect the voltage sensor across the resistor and the inductor.
   (d) Set the scope to watch each potential difference.
   (e) Set the waveform of the signal generator to Positive Square Wave.
   (f) Set the frequency and amplitude of the signal generator to 500 Hz and 1 V. Also check Auto of the signal generator.
   (g) Set the scope to watch each potential difference.
   (h) Set the trigger of the scope.
   (i) Select Fast Monitor Mode and click Monitor to observe.
   (j) Open the graph and plot the potential difference across the resistor and the inductor.
   (k) Click Highlight range of points in active data and fit the data to
   $$A + Be^{(-t/\tau)}.$$

2. **E2: a sine wave input**
   (a) Carry out experiments with a circuit on the board RL.
   (b) Connect the signal generator to V_{in} and the ground of the board.
   (c) Connect the voltage sensor across the ground and V_{in}. Also connect the voltage sensor across the resistor and the inductor.
   (d) Set the scope to watch each potential difference.
   (e) Set the waveform of the signal generator to Sine.
   (f) Set the frequency and amplitude of the signal generator to 500 Hz and 1 V. Also check Auto of the signal generator.
   (g) Set the scope to watch each potential difference.
   (h) Set the trigger of the scope.
   (i) Select Fast Monitor Mode and click Monitor to observe.
   (j) Measure the phase difference between v_R and v_L.

References

[1] ***KPOP*$\mathscr{E}$ Digital Library:**

[2] **Y14.7:** *Inductors and Inductance*

[3] **Y14.12:** *RL circuit*

[4] **Y16:** *Phase Shift in RLC Circuit*