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Team No. : __________ Date : __________ Lecturer's Signature : __________

Introduction

Goals

- Measure the capacitive time constant (τ_{RC}).
- Understand the capacitive reactance (X_C).
- Understand the phase shift between current and voltage.

Theoretical Backgrounds

1. **Potential Difference**
   (a) The potential difference V_R between the two terminals of a resistor of resistance R is
   $$V_R = RI,$$
   where I is the current, $I = \dfrac{dq}{dt}$.
   (b) The potential difference V_C between the two terminals of a capacitor of capacitance C charged with $+q$ and $-q$ is
   $$V_C = \frac{q}{C}.$$
2. **Resistance**: Consider a resistor made of a uniform material of resistivity ρ with the cross sectional area A and length ℓ.
   (a) The resistance is
   $$R = \rho\frac{\ell}{A}.$$
   (b) It is straightforward to derive the formula for the equivalent resistance for a serial connection of N resistors R_i's:
   $$R = \sum_{i=1}^{N} R_i.$$
   (c) It is straightforward to derive the formula for the equivalent resistance for a parallel connection of N resistors R_i's:
   $$\frac{1}{R} = \sum_{i=1}^{N} \frac{1}{R_i}.$$
3. **Capacitance**: Consider a parallel plate capacitor of the cross sectional area A and distance d.
   (a) The capacitance is
   $$C = \varepsilon_0\frac{A}{d},$$
   where ε_0 is the permittivity constant for the vacuum.
   (b) It is straightforward to derive the formula for the equivalent capacitance for a parallel connection of N capacitors C_i's:
   $$C = \sum_{i=1}^{N} C_i.$$
   (c) It is straightforward to derive the formula for the equivalent resistance for a serial connection of N capacitors C_i's:
   $$\frac{1}{C} = \sum_{i=1}^{N} \frac{1}{C_i}.$$
4. **Kirchhoff's Rule**: The algebraic sum of the changes in potential during a travel over a closed loop vanishes.
   (a) The formula is
   $$\sum_{i=1}^{N} \Delta V_i = 0,$$
   where i indicates each circuit element on that closed path.
   (b) The value is not vanishing if there is a changing magnetic field through the circuit that is not considered in this experiment.

5. ***RC* circuit with a square wave input**

   (a) This is the first-order linear differential equation:

   $$v_R + v_C = R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E},$$

   where $\mathscr{E}$ is the potential difference between the two terminals of the battery.

   (b) ***Charging***: If $q(t=0) = 0$, then

   $$\begin{aligned} v_C(t) &= \frac{q}{C} = \mathscr{E}(1 - e^{-t/\tau_{\mathrm{RC}}}), \\ v_R(t) &= Ri = \mathscr{E}e^{-t/\tau_{\mathrm{RC}}}, \end{aligned}$$

   where τ_{RC} is the ***time constant***,

   $$\tau_{\mathrm{RC}} = RC.$$

   (c) ***Discharging***: If $q(t=0) = q_0 = CV_0$ and $\mathscr{E} = 0$, then

   $$\begin{aligned} v_C(t) &= \frac{q}{C} = V_0 e^{-t/\tau_{\mathrm{RC}}}, \\ v_R(t) &= RI = -V_0 e^{-t/\tau_{\mathrm{RC}}}. \end{aligned}$$

6. ***RC* circuit with a sine wave input**

   (a) This is the first-order linear differential equation with a sine wave input:

   $$v_R + v_C = R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E}_m \sin\omega_{\mathrm{d}} t,$$

   where ω_{d} ($\mathscr{E}_m$) is the frequency (amplitude) of the sine wave input.

   (b) The current $i(t)$ of the circuit is

   $$i(t) = \frac{\mathscr{E}_m}{Z}\sin(\omega_{\mathrm{d}} t - \phi),$$

   where Z is

   $$Z = \sqrt{R^2 + X_C^2},$$

   X_C is the ***capacitive reactance***

   $$X_C = \frac{1}{\omega_{\mathrm{d}} C},$$

   and ϕ is the ***phase constant***

   $$\phi = \arctan\left(-\frac{X_C}{R}\right).$$

   (c) The potential difference $v_R(t)$ of R is

   $$\begin{aligned} v_R(t) &= i(t)R \\ &= \frac{\mathscr{E}_m R}{Z}\sin(\omega_{\mathrm{d}} t - \phi). \end{aligned}$$

   The phase of $i(t)$ and that of $v_R(t)$ are **in phase**.

   (d) The potential difference $v_C(t)$ of C is

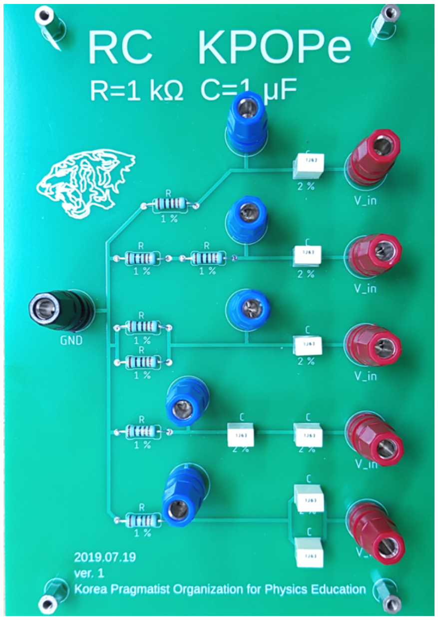
   $$\begin{aligned} v_C(t) &= \frac{q(t)}{C} \\ &= \frac{1}{C}\int i(t)dt \\ &= -\frac{\mathscr{E}_m}{Z\omega_{\mathrm{d}} C}\cos(\omega_{\mathrm{d}} t - \phi) \\ &= \frac{\mathscr{E}_m}{Z\omega_{\mathrm{d}} C}\sin(\omega_{\mathrm{d}} t - \phi - \frac{\pi}{2}). \end{aligned}$$

   The phase of $i(t)$ **leads** that of $v_C(t)$ by $\frac{\pi}{2}$, or the phase of $v_C(t)$ **lags** that of $i(t)$ by $\frac{\pi}{2}$.

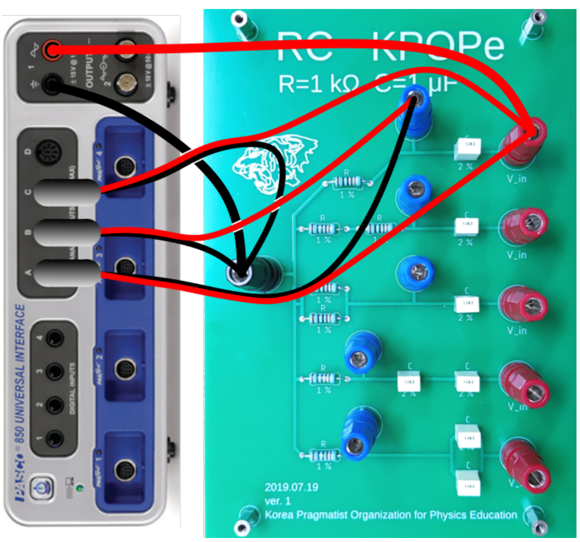
   (e) The phase of $v_R(t)$ **leads** that of $v_C(t)$ by $\frac{\pi}{2}$, or the phase of $v_C(t)$ **lags** that of $v_R(t)$ by $\frac{\pi}{2}$.

Instrumentation

1. **Board *RC***



2. **Connection**



Experimental Procedure

1. **E1: a square wave input**
   (a) Carry out experiments with the five circuits on the board *RC*.
   (b) Connect the signal generator to V_{in} and the ground of the board.
   (c) Connect the voltage sensor across V_{in} and the ground.
   (d) Connect the voltage sensor across the resistor and the capacitor.
   (e) Set the waveform of the signal generator to Positive Square Wave.
   (f) Set the frequency and amplitude of the signal generator to 50 Hz and 1 V. Also check Auto of the signal generator.
   (g) Set the scope to watch each potential difference.
   (h) Set the trigger of the scope.
   (i) Select Fast Monitor Mode and click Monitor to observe.
   (j) Open the graph and plot the potential difference across the resistor and the capacitor.
   (k) Click Highlight range of points in active data and fit the data to

$$A + Be^{(-t/\tau)}.$$

2. **E2: a sine wave input**
   (a) Carry out experiment with a circuit on the board *RC*.
   (b) Connect the signal generator to V_{in} and the ground of the board.
   (c) Connect the voltage sensor across V_{in} and the ground.
   (d) Connect the voltage sensor across the resistor and the capacitor.
   (e) Set the waveform of the signal generator to Sine.
   (f) Set the frequency and amplitude of the signal generator to 50 Hz and 1 V. Also check Auto of the signal generator.
   (g) Set the scope to watch each potential difference.
   (h) Set the trigger of the scope.
   (i) Select Fast Monitor Mode and click Monitor to observe.
   (j) Measure the phase difference between v_R and v_c.

References

[1] ***KPOP*ℰ Digital Library:**

[2] **Y06:** *Capacitance*

[3] **Y09:** *RC Circuit*

[4] **Y16:** *Phase Shift in RLC Circuit*