Name: _____ Department: _____ Student ID: _____

Team No.: _____ Date: _____ Lecturer's Signature: _____

Introduction

Goal

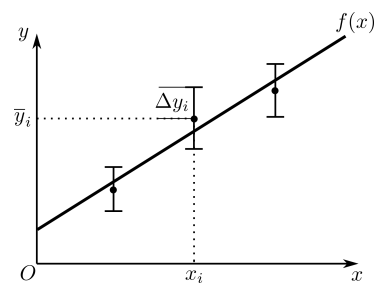
- Introducing χ^2 method as an example of the goodness of fit test.

Theoretical Backgrounds

1. **Measurement and Errors**
When we measure a physical observable like the length L of an object, the measured values, which are real numbers in units of the corresponding physical dimensions, are not always equal because the measurement accompanies ***uncertainties***, i.e. ***errors***.

2. **Average and Standard Deviation**
Consider a physical variable y that depends on the control variable x. We select N points x_i and the corresponding y value is y_i. For each x_i, we repeat the measurement of y_i K times and the values are y_{ik} for $k = 1, \cdots, K$. Then the true value y_i can be estimated within $\overline{y}_i - \Delta y_i \leq y_i \leq \overline{y}_i + \Delta y_i$:
$$y_i = \overline{y}_i \pm \Delta y_i,$$
where $\overline{y}_i$ is the ***mean*** (***average***) value and Δy_i is the ***uncertainty***:
$$\begin{aligned} \overline{y}_i &\equiv \frac{1}{K}\sum_{k=1}^{K} y_{ik}, \\ \Delta y_i &= \frac{\sigma_i}{\sqrt{K}} \equiv \sqrt{\frac{1}{K(K-1)}\sum_{k=1}^{K}(y_{ik}-\overline{y}_i)^2}. \end{aligned}$$
Here, σ_i is the unbiased ***standard deviation***
$$\sigma_i = \sqrt{\frac{1}{K-1}\sum_{k=1}^{K}(y_{ik}-\overline{y}_i)^2}.$$
For example, suppose that you have measured the length y_1 of a pen 5 times as $y_{11} = 12.8\,\text{cm}$, $y_{12} = 12.6\,\text{cm}$, $y_{13} = 12.8\,\text{cm}$, $y_{14} = 12.7\,\text{cm}$, $y_{15} = 12.5\,\text{cm}$.
   (a) The average is $\overline{y}_1 = 12.7\,\text{cm}$.
   (b) The uncertainty is $\Delta y_1 = 0.1\,\text{cm}$.

3. **χ^2**
We introduce a model function $y = f(x)$ that reproduces the data as $y_i \approx f(x_i)$. In order to find the best choice of $f(x)$, we minimize χ^2 that is defined by
$$\chi^2 = \sum_{i=1}^{N}\left[\frac{\overline{y}_i - f(x_i)}{\Delta y_i}\right]^2.$$
   - Model parameters of $f(x)$ are to be determined when they minimize χ^2.



   - When the model fits the data well, $\chi^2 \to$ the number of degrees of freedom $N - M$. Here, N is the number of random variables and M is the number of model parameters in $y = f(x)$.

For example, suppose that you have measured the trajectory of a particle moving along the y axis at time t_i's. We introduce a model function $y = f(t) = vt$.

i		1	2	3	4	5
t_i	(s)	1.0	2.0	3.0	4.0	5.0
$\overline{y}_i$	(cm)	2.1	3.9	5.8	8.2	10.1
Δy_i	(cm)	0.1	0.1	0.1	0.1	0.1

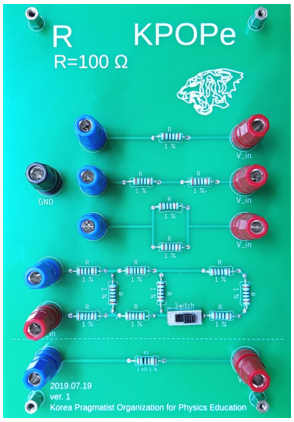
   (a) We define a dimensionless variable $a = v \times \text{s/cm}$. Let us express χ^2 in terms of a. Then we find that
$$\chi^2 = 5500a^2 - 22120a + 22251.$$

©2023 ***KPOPℰ*** All rights reserved.

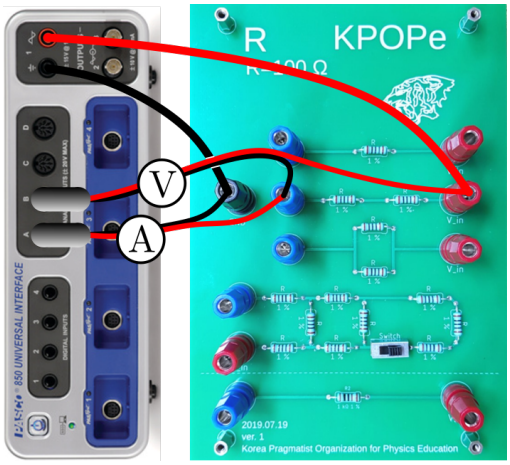
(b) We can determine the parameter a that minimizes χ^2. The minimum value for the χ^2 is 10 at $v = 2.0$ cm/s.

Instrumentation

1. **Board *R***
   (a) the single R circuit
   (b) the serial R circuit
   (c) the parallel R circuit
   (d) the circuit with a switch (off)
   (e) the circuit with a switch (on)



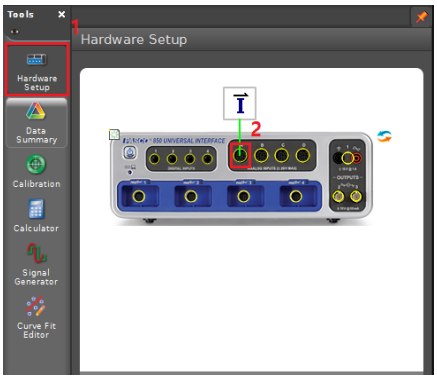
2. **Connection**
   (a) Connect the *PASCO Interface 850* device to the board R as shown in the following figure.



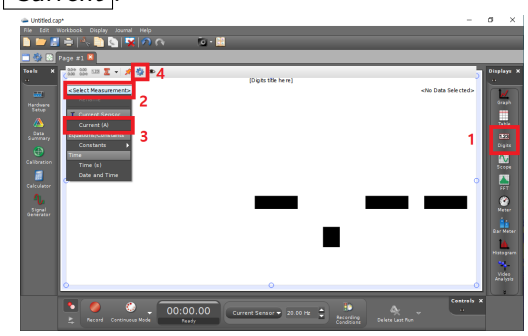
©2023 ***KPOPe*** All rights reserved.

Experimental Procedure

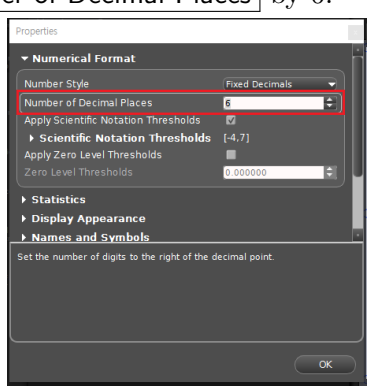
1. **E1:** Direct Current (DC) Experiment
   (a) Carry out an experiment with the single R circuit with $R = 100\ \Omega$, as instructed in below steps.
   (b) Select Current Sensor as shown in the following figure.



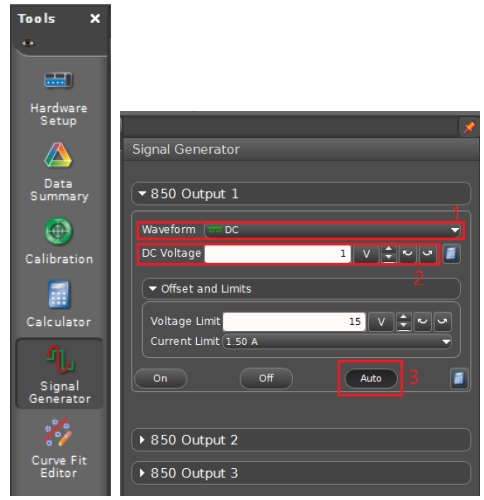
   (c) Measure the value of current I_R with in units of A to the sixth digit below the decimal point.
   - Click Digits, Select Measurement, and Current.



   - Click Properties and set Number of Decimal Places by 6.

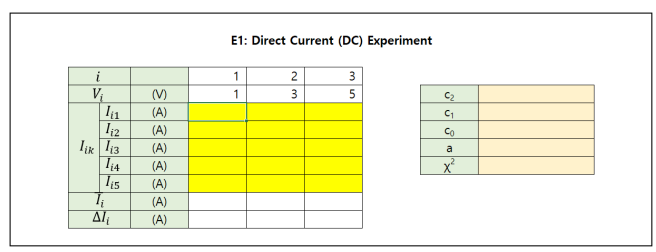


   (d) Apply the DC to the circuit: click Signal Generator and set parameters as shown in the following figure.



   (e) Set Fast Monitor Mode and click Monitor to measure the current.

   (f) Open the *Excel* file. Fill the yellow boxes on the *Excel* sheet with measured values.



E1: Direct Current (DC) Experiment

i		1	2	3
V_i	(V)	1	3	5
I_{ik} I_{i1}	(A)			
I_{i2}	(A)			
I_{i3}	(A)			
I_{i4}	(A)			
I_{i5}	(A)			
I_i	(A)			
ΔI_i	(A)			

c_2	
c_1	
c_0	
a	
χ^2	

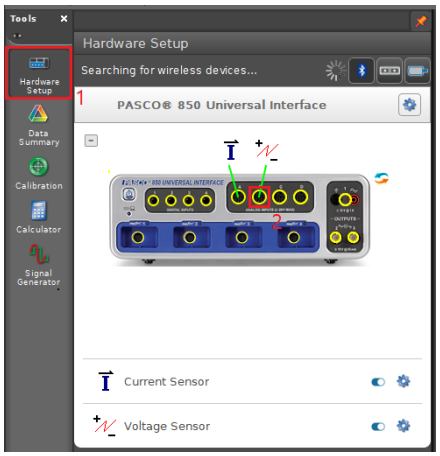
©2023 ***KPOPℰ*** All rights reserved.

2. **E2:** Alternating Current (AC) Experiment

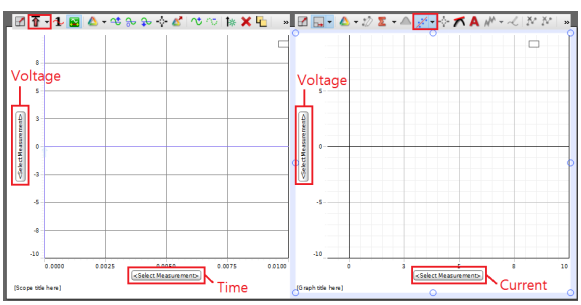
   (a) Carry out experiments with the five circuits on the borad R.

   (b) Scope / Graph setup

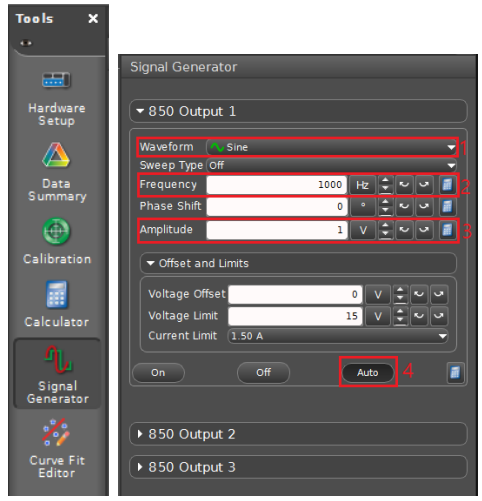
   - Select Current Sensor and Voltage Sensor.



   - Click Scope and Graph and display both windows.
   - Set each axis as shown in the following figure.

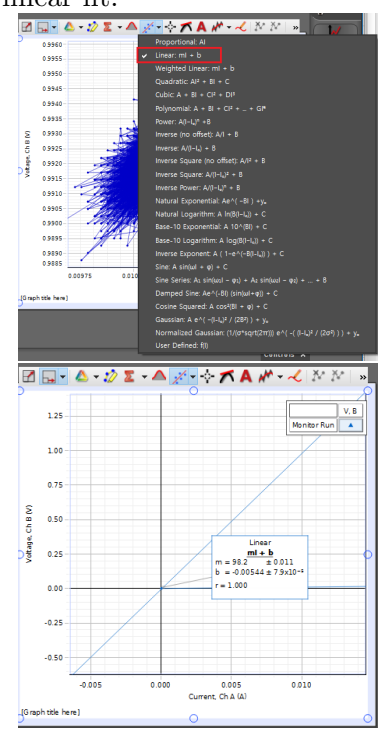


   (c) Apply the AC to the circuit: click Signal Generator and set parameters as shown in the following figure.



   (d) Set Fast Monitor Mode and click Monitor to get data.

   (e) Draw a V_R-I_R plot in the Graph window and make a linear fit.



   (f) Take screen shots of the fitting results and save the images into the *Excel* file.

©2023 ***KPOPℰ*** All rights reserved.

References

[1] ***KPOPℰ* Digital Library:**

[2] **ESA23/A01:** *Numbers in Physics differ from those in Mathmatics*

[3] **ESA23/A03:** *Algebra of Physical Measurables*

[4] **Y07:** *Current and Resistance*

©2023 ***KPOPℰ*** All rights reserved.