Name: ______________ Department: ______________ Student ID: ______________

Team No. : ______________ Date : ______________ Lecturer's Signature : ______________

Introduction

1. Longitudinal Wave

Goals

- Understand the nature of the longitudinal wave with sound waves.
- Detect the nodes or antinodes of a longitudinal standing wave.
- Experience the sound-cancellation phenomenon.
- Estimate the speed of sound in the air.

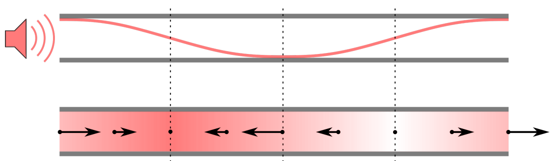


Fig. 1: The first overtone of the standing-wave resonance emerging inside a pipe with both ends open. The red curve inside the pipe represents the amplitude of the standing wave. There are three antinodes at both ends and at the center. The lower figure represents the density or pressure of the air molecules per unit volume: the darker the color, the greater the density or pressure; the longer the arrow, the larger the amplitude of the longitudinal oscillation of air molecules.

Theoretical Backgrounds

(a) The sound wave is a compressional wave along the longitudinal direction. When it travels in the air as a medium, the pressure oscillates back and forth along the longitudinal direction, the direction of wave propagation.

(b) The speed v of the sound wave in fluids is given by the Newton-Laplace equation:

$$v = \sqrt{\frac{B}{\rho}}, \qquad (1)$$

where ρ is the mass density of the medium per volume and B is the bulk modulus defined by

$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta p}{\Delta V/V} = \rho v^2. \qquad (2)$$

Here, Δp and ΔV are the corresponding changes in the pressure p and volume V.

(c) The oscillation of the pressure at x at time t can be described by the average value of the longitudinal displacement of molecules at that space-time point as

$$s(t,x) = s_m \cos(kx - \omega t),$$

where s_m is the displacement amplitude from equilibrium position and we have neglected the constant phase.

(d) If the cross-sectional area of the pipe is A, then

- $V = A\Delta x$ is the volume of the air sample in $[x, x+\Delta x]$,
- $\Delta V = A\Delta s$ is the change in V,

According to (2), the pressure change Δp due to the sound wave is given by

$$\Delta p = -B\frac{\Delta V}{V} = -\rho v^2 \frac{\Delta s}{\Delta x}.$$

In the limit as $\Delta x \to 0$,

$$\begin{aligned} \Delta p &= -\rho v^2 \frac{\partial s}{\partial x} \\ &= (\rho v \omega) s_m \sin(kx - \omega t), \end{aligned} \qquad (3)$$

where note that $k = \omega/v$.

(e) A **pressure microphone** *is a device that detects the pressure change '*$\Delta p(t,x)$*' instead of the displacement '*$s(t,x)$*'*. It is realized by exposing the diaphragm on only one side to the impinging sound. When measuring the amplitude of a sound wave using a pressure microphone, the greater the signal, the smaller the longitudinal displacement $s(t,x)$.

(f) At certain frequencies, standing waves occur as the harmonic series in pipes of length L. The emergence of the standing waves indicates that such a sound wave is a resultant wave of two identical waves propagating in opposite directions:

$$\begin{aligned} s(t,x) &= s_m \cos(kx-\omega t) + s_m \cos(kx+\omega t) \\ &= [2s_m \cos kx] \cos \omega t, \qquad (4) \\ \Delta p(t,x) &= a_m \sin(kx-\omega t) + a_m \sin(kx+\omega t) \\ &= [2a_m \sin kx] \cos \omega t, \qquad (5) \end{aligned}$$

where $a_m = v\rho\omega s_m$.

At an **open end**:

- The x-dependent factor in the displacement s has the maximum value.
- The x-dependent factor in the pressure change Δp has the minimum value.

At a **closed end**:

- The x-dependent factor in the displacement s has the minimum value.
- The x-dependent factor in the pressure change Δp has the maximum value.

(g) A **speaker** generates a sound wave in such a way that the vibration of the diaphragm derives the displacement $s(t,x)$ of air molecules. Suppose that a standing wave emerges due to the driving force of the vibrating speaker placed at $x = x_0$.

- x_0 must be the antinode of the standing wave for $s(t,x)$.
- x_0 must be the node of the standing wave for $\Delta p(t,x)$:

The same is true for any open end. In summary, if there is an **open end at** 'x_0,' then

$$\frac{\partial}{\partial x} s(t,x_0) = 0, \qquad (6a)$$

$$\Delta p(t,x_0) = 0. \qquad (6b)$$

(h) A **closed end** of a pipe reflects a sound wave in such a way that the displacement $s(t,x)$ of air molecules is vanishing on the wall. Suppose that a standing wave emerges inside a pipe and a closed end (wall) is placed at $x = x_0$.

- x_0 must be the node of the standing wave for $s(t,x)$.
- x_0 must be the antinode of the standing wave for $\Delta p(t,x)$.

In summary, if there is a **closed end at** 'x_0,' then

$$s(t,x_0) = 0, \qquad (7a)$$

$$\frac{\partial}{\partial x} \Delta p(t,x_0) = 0. \qquad (7b)$$

2. **E1: Two Open Ends**

Goals

- Understand the resonance in the pipe of two open ends.
- Identify the nodes and antinodes for the pressure change $\Delta p(t,x)$.
- Identify the nodes and antinodes for the displacement $s(t,x)$.
- Estimate the speed of sound in the air.

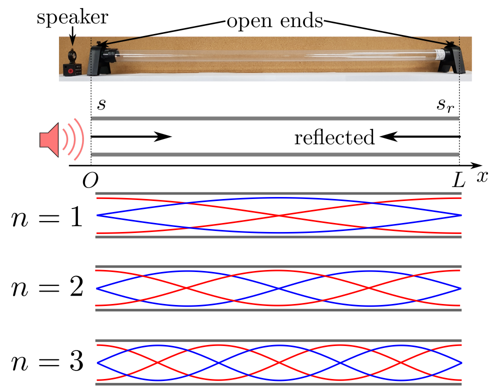


Fig. 2: The three lowest frequencies of the harmonics inside a pipe with two open ends. The red and blue curves represent $s(t,x)$ and $\Delta p(t,x)$, respectively.

Theoretical Backgrounds

(a) The coordinates for the both ends are $x = 0$ and L. According to (5) and the boundary condition for an open end (6), we find that

$$\sin k0 = \sin kL = 0. \qquad (8)$$

The solution is $kL = n\pi$ for any positive integer n. Substituting $k = 2\pi/\lambda$, we find the quantized values for the wavelength as

$$\lambda_n = \frac{2L}{n}, \quad n = 1,\, 2,\, 3,\, \cdots .$$

(b) The corresponding quantized values for the resonant frequencies that generate standing waves are

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}v, \quad n = 1, 2, 3, \cdots .$$

3. **E2: One Open, the Other Closed**

Goals

- Understand the resonance in the pipe with an open end and a closed one.
- Identify the nodes and antinodes for the pressure change $\Delta p(t, x)$.
- Identify the nodes and antinodes for the displacement $s(t, x)$.
- Estimate the speed of sound in the air.

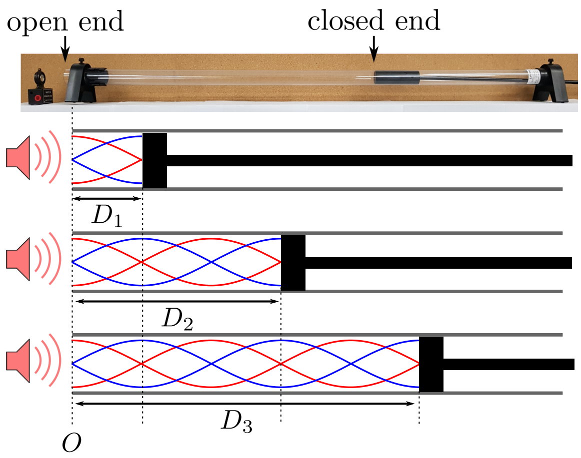


Fig. 3: The three lowest D_m's defined in (9) for a sound wave of frequency f inside a pipe whose left end is open and the other end is closed . The red and blue curves represent the x dependences of the resultant waves for $s(t, x)$ and $\Delta p(t, x)$, respectively.

Theoretical Backgrounds

(a) The coordinates for the open and closed ends are $x = 0$ and L, respectively. According to (5) and the boundary condition for an open end (6), we find that

$$\sin k0 = 0.$$

This is a trivial equation. According to (5) and the boundary condition for an open end (7), we find that

$$\cos kL = 0.$$

The solution is

$$kL = n\pi - \frac{\pi}{2}$$

for any integer n. Substituting $k = 2\pi/\lambda$, we find the quantized values for the wavelength as

$$\lambda_n = \frac{2L}{n - \frac{1}{2}}, \quad n = 1, 2, 3, \cdots .$$

(b) The corresponding quantized values for the resonant frequencies that generate standing waves are

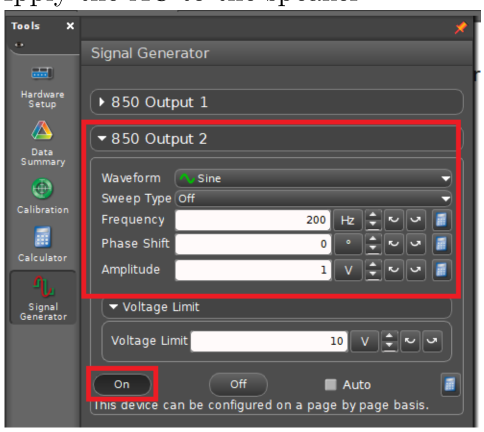
$$f_n = \frac{v}{\lambda_n} = \frac{n - \frac{1}{2}}{2L}v, \quad n = 1, 2, 3, \cdots .$$

(c) If we keep the frequency invariant, then the standing waves emerge for the following values D_m for the variable length L:

$$D_m = \frac{\lambda}{2}\left(m - \frac{1}{2}\right), \quad m = 1, 2, 3, \cdots . \tag{9}$$

Experimental Procedure

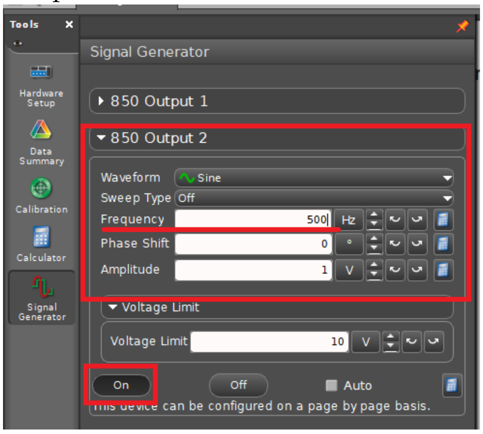
1. **E1**
   (a) Measure the length of a pipe.
   (b) Locate a speaker at the end of the pipe and connect the speaker to output 2 of a *PASCO 850 Interface*.
   (c) Apply the AC to the speaker



   (d) Find the resonant frequencies when n=1, 2, and 3.

2. **E2**
   (a) Locate a speaker at the end of the pipe and connect the speaker to output 2 of the interface.

(b) Apply the AC with frequency 500 Hz to the speaker.



(c) Inset a piston to the other end of the pipe.

(d) Move the piston and measure the length between open end and piston when resonance.

Appendix 1: Noise Cancelling

Goals

- Investigate the interference pattern of two sound sources.
- Understand the mechanism of the noise cancellation.

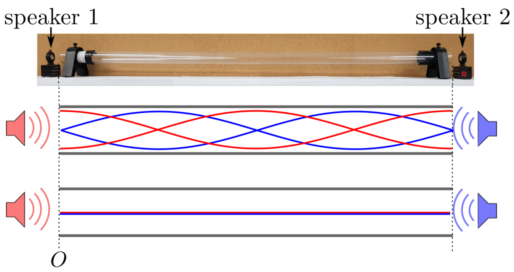


Fig. 4: When the frequency of two sound sources S_1 and S_2, is a resonant frequency, the resultant wave can be analyzed as a superposition of two standing waves. The first (second) figure shows the case where the phase difference is π (0) between the two standing waves in the second harmonic ($n = 2$). Here, the red and blue curves describe the x dependences of the resultant waves for $s(t, x)$ and $\Delta p(t, x)$, respectively.

Theoretical Backgrounds

(a) The two identical sound sources S_1 and S_2 are placed at one end and the other end, respectively, of the pipe with both ends open. If they generate a standing wave of equal amplitude, then the displacements $s_i(t, x)$ induced by S_i are given by

$$\begin{aligned} s_1(t,x) &= 2s_m \cos kx \cos \omega t, \\ s_2(t,x) &= -2s_m \cos[k(L-x)] \cos(\omega t + \delta) \\ &= (-1)^{n+1} \times 2s_m \cos kx \cos(\omega t + \delta), \end{aligned}$$

where n represents the nth harmonic of the standing waves. Here, δ is the relative phase between two sources which is not zero in general. Note that the overall (-1) in the second line in $s_2(t, x)$ is due to that the direction of the oscillations of air molecules is flipped with respect to $s_1(t, x)$. In the last line, the following identity has been used:

$$\begin{aligned} \cos[k(L-x)] &= \cos\left(n\pi - \frac{n\pi x}{L}\right) \\ &= \cos n\pi \cos \frac{n\pi x}{L} \\ &= (-1)^n \cos kx. \end{aligned}$$

(b) By making use of the following trigonometric identities:

$$\begin{aligned} \cos(\omega t) &= \cos(\omega t + \frac{\delta}{2}) \cos \frac{\delta}{2} \\ &\quad + \sin(\omega t + \frac{\delta}{2}) \sin \frac{\delta}{2}, \\ \cos(\omega t + \delta) &= \cos(\omega t + \frac{\delta}{2}) \cos \frac{\delta}{2} \\ &\quad - \sin(\omega t + \frac{\delta}{2}) \sin \frac{\delta}{2}, \end{aligned}$$

we simply the resultant displacement as

$$s_1 + s_2 = \begin{cases} 4s_m \cos kx \cos\left(\omega t + \frac{\delta}{2}\right) \cos \frac{\delta}{2}, \text{ for odd } n, \\ 4s_m \cos kx \sin\left(\omega t + \frac{\delta}{2}\right) \sin \frac{\delta}{2}, \text{ for even } n. \end{cases} \tag{10}$$

(c) The resultant sound completely disappears for any x and t if

$$\begin{cases} \cos \frac{\delta}{2} = 0, \text{ for odd } n, \\ \sin \frac{\delta}{2} = 0, \text{ for even } n, \end{cases}$$

or, equivalently,

$$\delta = \begin{cases} (2m+1)\pi, \text{ for odd } n, \\ 2m\pi, \text{ for even } n, \end{cases} \tag{11}$$

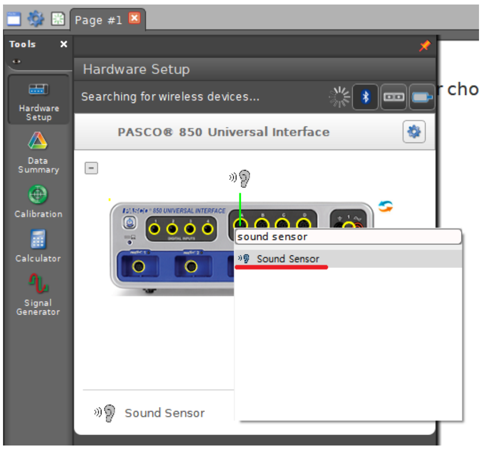
where m is an integer.

Experimental Procedure

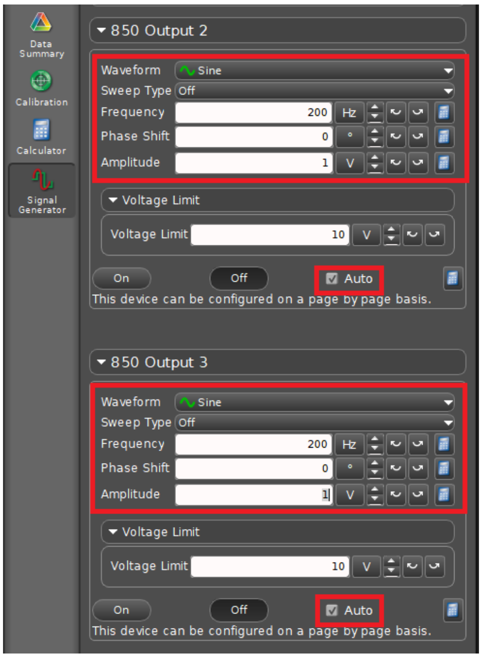
(a) Locate two identical speakers to the ends of a pipe and connect the speakers to output 2 and 3 of the interface.

(b) Locate a microphone inside of the tube.

(c) Connect the microphone to the analog input A and set Sound Sensor.

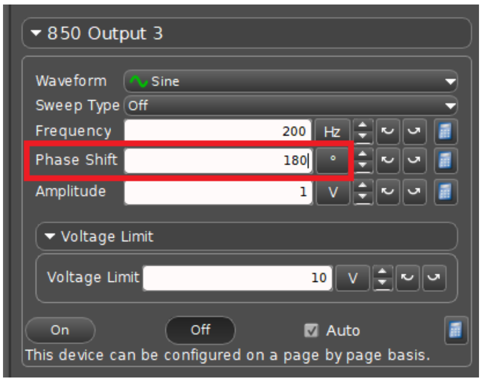


(d) Apply the identical AC to both speakers with frequency measured at **E1**.



(e) Set Fast Monitor Mode and click Monitor to collect data.

(f) Change the phase shift of OUTPUT 3 to 180°.



(g) Draw the $V(t)$ graph and check the amplitudes of in phase and out of phase.

(h) Repeat steps (d)–(g) after replacing the frequency.

Appendix 2: Beats

Goals

- Measure the pressure change $\Delta p(t)$ of the air using a microphone.
- Estimate the beat frequency $f_{\rm beat}$.

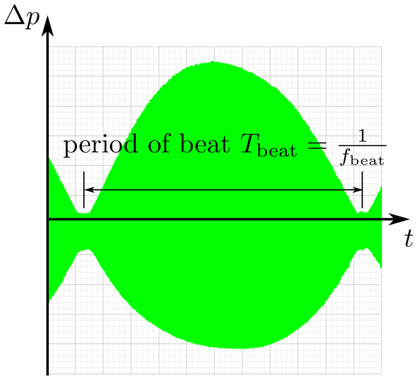


Fig. 5: If the frequencies of S_1 and S_2 are slightly different, then there is a sharp variation in the intensity of the sound from overlapping s_1 and s_2. The intensity of the sound increases and dereases in slow, wavering beats that repeat at a frequency of $f_{\rm beat}$.

Theoretical Backgrounds

(a) Suppose that we prepare a sound source S_1 at $x = 0$ of a pipe with both ends open and adjust the frequency at a resonance frequency

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}v, \quad n = 1, 2, \cdots .$$

Then, the displacement s_1 induced by S_1 is given by

$$s_1(t, x) = 2s_m \cos kx \cos \omega t,$$

where

$$\omega = 2\pi f_n, \quad k = \frac{n\pi}{L}.$$

The other sound source S_2 is placed at the other end of the pipe, $x = L$, which generates the sound wave of slightly different frequency f'_n from S_1 by

$$f'_n = f_n(1 + \delta), \quad \delta \ll 1.$$

The displacement s_2 induced by S_2 is a sum of the incident wave and its reflected wave:

$$\begin{aligned} s_2(t, x) &= -s'_m \cos[k'(L - x) - \omega' t] \\ &\quad -s'_m \cos[k'x - \omega'(t - L/v)] \\ &= -s'_m \cos(k'x + \omega' t - k'L) \\ &\quad -s'_m \cos(k'x - \omega' t + k'L) \\ &= -2s'_m \cos k'x \cos(\omega' t - k'L) \\ &= (-1)^{n+1} 2s'_m \cos k'x \cos(\omega' t - \phi), \end{aligned}$$

where $\phi = \delta n\pi$,

$$\omega' = 2\pi f'_n = 2\pi f(1 + \delta), \; k' = \frac{n\pi}{L}(1 + \delta).$$

Here, S_1 and S_2 share the same medium so that

$$v = \frac{\omega}{k} = \frac{\omega'}{k'}.$$

Note that the following trigonometric identity has been used:

$$\cos A + \cos B = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right).$$

(b) The pressure changes Δp_1 and Δp_2 due to S_1 and S_2, respectively, are given by

$$\begin{aligned} \Delta p_1 &= 2s_m k \sin kx \cos \omega t, \\ \Delta p_2 &= (-1)^{n+1} 2s'_m k' \sin k'x \cos(\omega' t - \phi). \end{aligned}$$

(c) Assume that we take $x = x_0$ that satisfies the following condition:

$$2s_m k \sin kx_0 = (-1)^{n+1} 2s'_m k' \sin k'x \equiv A.$$

At the point $x = x_0$, the total pressure change $\Delta p = \Delta p_1 + \Delta p_2$ is given by

$$\begin{aligned} \Delta p &= A\cos \omega t + A\cos(\omega' t - \phi) \\ &= \left[2A\cos\left(\frac{\omega - \omega'}{2}t + \frac{\phi}{2}\right)\right]\cos\left(\frac{\omega + \omega'}{2}t - \frac{\phi}{2}\right). \end{aligned}$$

Note that the phase difference $\phi/2$ does not affect the time dependence of Δp.

The amplitude of Δp becomes maximum when $\cos\left(\frac{\omega - \omega'}{2}t + \frac{\phi}{2}\right) = \pm 1$. Therefore, the beat angular frequency $\omega_{\rm beat}$ is given by

$$\omega_{\rm beat} = |\omega - \omega'|.$$

The corresponding frequency $f_{\rm beat}$ is

$$f_{\rm beat} = |f_n - f'_n|.$$

Experimental Procedure

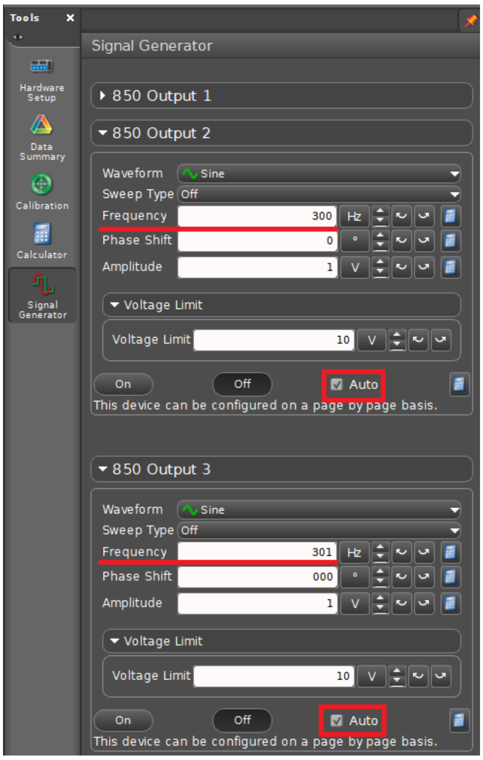
(a) Locate two identical speakers to the ends of a pipe and connect the speakers to output 2 and 3 of the interface.

(b) Locate a microphone inside of the tube.

(c) Connect the microphone to the analog input A and set Sound Sensor.

(d) Set gain of the sound sensor to 100x in sound sensor properties.

(e) Apply the AC to the speakers with small frequency difference.



(f) Draw the $V(t)$ graph and measure the frequency of beating.

(g) Repeat steps (e)–(f) after replacing frequencies of the speakers.