Name: ____________ Department: ____________ Student ID: ____________

Team No. : ____________ Date : ____________ Lecturer's Signature : ____________

Introduction

1. **Standing Waves on A String**

Goals

- Understand the transverse wave.
- Understand the superposition principle.
- Observe standing waves on a string.
- Estimate the wave speed on the string.

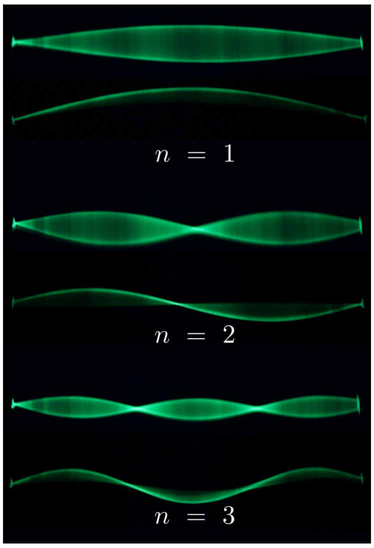


Figure 1: Standing wave patterns on a string: the first harmonic (top), the second harmonic (middle), and the third harmonic (bottom).

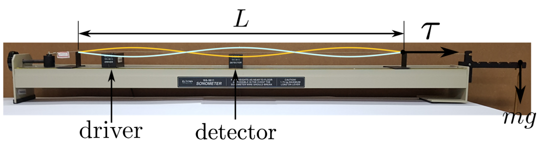


Figure 2: A sonometer system. At certain driving frequencies we observe standing waves.

Theoretical Backgrounds

Consider a transverse vibration along the y axis of a string stretched along the x (longitudinal) axis. For simplicity, we restrict ourselves to the vibration along the y axis and neglect that along the z axis which is actually allowed in nature. We also ignore the vibration along the longitudinal direction.

(a) A wave is called transverse if the vibration is perpendicular to the longitudinal direction along which the wave propagates.

(b) A massive string of the linear mass density ρ with a tension τ is a carrier of a propagating wave.

(c) The speed of the wave propagation along a vibrating string

$$v = \sqrt{\frac{\tau}{\rho}} \tag{1}$$

was discovered in the late 1500s by Vincenzo Galilei, the father of Galileo Galilei.

(d) A transverse wave is described by a time-dependent field $y(t,x)$, a function of both x and t. Here, $y(t,x)$ is the transverse displacement of a tiny segment of the string at x.

(e) A wave propagating along the x axis with the propagating speed v is a solution to the wave equation:

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}y(t,x) = \frac{\partial^2}{\partial x^2}y(t,x). \tag{2}$$

(f) For any differentiable function $f(x)$, $y(t,x) = f(x \pm vt)$ is a wave satisfying the wave equation (2). Here, $f(x-vt)$ propagates along $+\hat{\mathbb{i}}$ and $f(x+vt)$ along $-\hat{\mathbb{i}}$.

(g) A sinusoidal function

$$y_\pm(t,x) = y_m \sin(kx \mp \omega t + \phi) \tag{3}$$

is a wave that propagates along $\pm\hat{\mathbb{i}}$ with the speed

$$v = \frac{\omega}{k}.$$

Here, ϕ is a constant phase, ω is the angular frequency and k is the wave number:

$$\omega = \frac{2\pi}{T}, \qquad k = \frac{2\pi}{\lambda},$$

where T and λ are the period and the wavelength, respectively.

(h) In summary, the wave speed can be expressed as

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f,$$

where $f = 1/T$ is the frequency.

(i) A postulate of the wave is that the resultant wave of any two waves is just the algebraic sum:

$$y(t,x) = y_1(t,x) + y_2(t,x).$$

(j) We consider the resultant wave of $y_+(t,x)$ and $y_-(t,x)$ that are defined in (3). These waves are of common amplitude and phase except that their directions of propagation are opposite:

$$\begin{aligned} y(t,x) &= y_+(t,x) + y_-(t,x) \\ &= y_m \sin(kx - \omega t + \phi) \\ &+ y_m \sin(kx + \omega t + \phi) \\ &= [2y_m \sin(kx + \phi)] \cos \omega t. \quad (4) \end{aligned}$$

(k) The resultant wave in (4) does not propagate because the x dependence and the t dependence are factorized (decoupled) completely. Thus the wave does not propagate but stands. Thus we call it a **standing wave**.

(l) The x dependence $\sin(kx + \phi)$ has zeros periodically at

$$x_{\text{node}} = \frac{n\pi - \phi}{k} = \left(\frac{n}{2} - \frac{\phi}{2\pi}\right)\lambda.$$

We call the points at which the standing wave always vanishes the **nodes**. The distance between the two nearest nodes is $\lambda/2$.

(m) The x dependence $\sin(kx + \phi)$ has the maximum value 1 periodically at

$$x_{\text{antinode}} = \frac{(n+\frac{1}{2})\pi - \phi}{k} = \left(\frac{n+\frac{1}{2}}{2} - \frac{\phi}{2\pi}\right)\lambda.$$

We call the points at which the standing wave always has the largest vibration the **antinodes**. The distance between the two nearest antinodes is $\lambda/2$.

(n) For a stretched string of length L with both ends fixed, the two ends must be nodes simultaneously. Therefore, the conditions for the standing waves are

$$L = \frac{\lambda}{2} n, \quad n = 1, 2, \cdots .$$

(o) Thus the driver of a sonometer must have the vibrating frequencies satisfying $\lambda_n = 2L/n$ to have standing waves emerge. Thus the wavelengths and frequencies are quantized as

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\rho}},$$

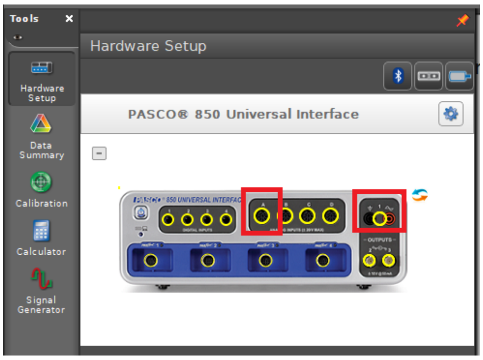
where the harmonic number n is a positive integer and λ_n is the wavelength of the wave at the nth harmonic. We call the lowest frequency (longest wavelength) of $n = 1$ state the **fundamental harmonic** or the **zeroth harmonic**. We call the higher frequency (shorter wavelength) than that of the fundamental harmonic the **overtones**. For example, $n = 2$ is the first overtone or the first harmonic.

Experimental Procedure

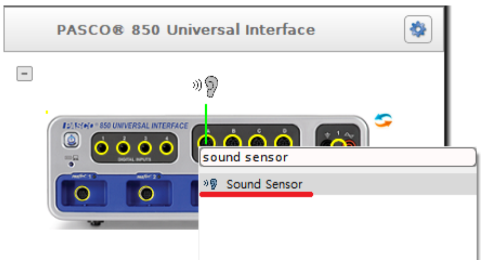
1. **E1**

(a) Connect a *PASCO Interface 850* device to a computer.

(b) Connect a Detector to ANALOG INPUT A of the interface.

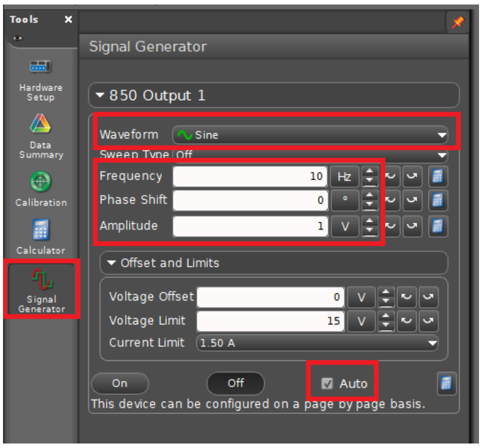


(c) Select Sound Sensor at Hardware Setup.

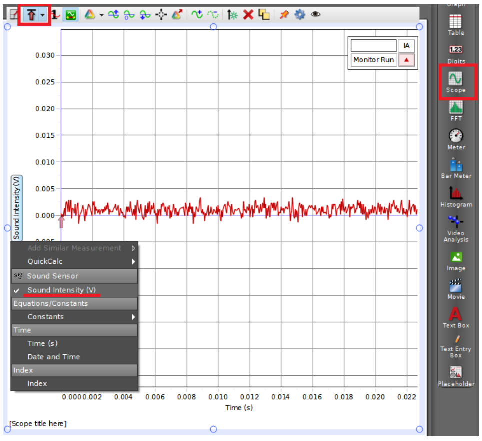


(d) Connect a Driver to OUTPUT 1 of the interface.

(e) Apply the AC to the Driver by Signal Generator.

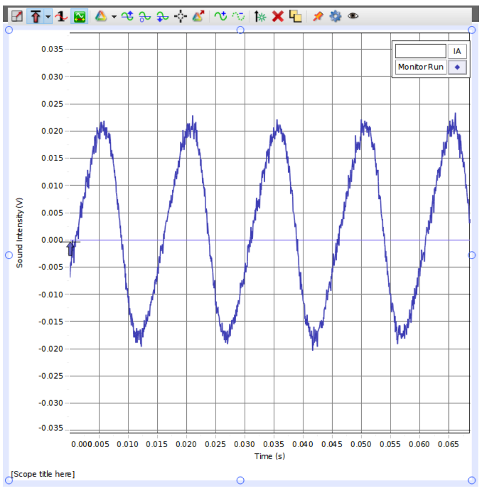


(f) Set the Scope to measure $V(t)$.



(g) Set Fast Monitor Mode and click Monitor to measure the intensity.

(h) Increase the frequency applied to the Driver and find the lowest frequency to maximize the amplitude.



- Note that the resonant frequency of the string is twice the frequency applied to the Driver.

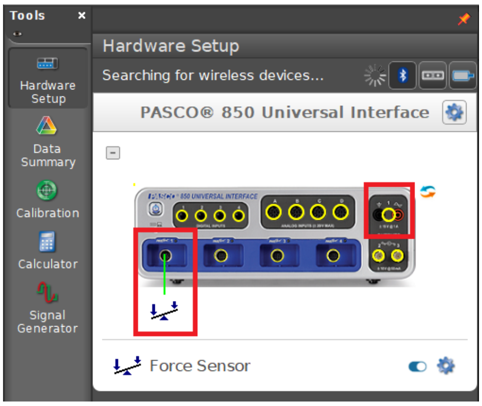
(i) Repeat the step (h) after replacing the density of the string.

(j) **The harmonic number** n **(only once during the A8)** : find the second and third lowest frequency to maximize the amplitude without replacing the string.

2. **E2**

(a) Connect a String Vibrator to OUTPUT 1 of the interface.

(b) Connect a Force Sensor to PASPORT 1 of the interface.



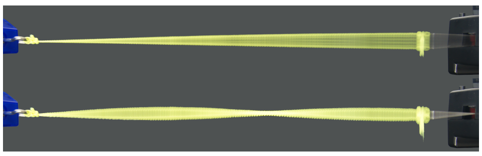
- The *PASCO Interface 850* will automatically recognize the Force Sensor.

(c) Set the Scope to measure $F(t)$.

(d) Apply the AC to the String Vibrator by Signal Generator.

(e) Set Fast Monitor Mode and click Monitor to measure the tension of the string.

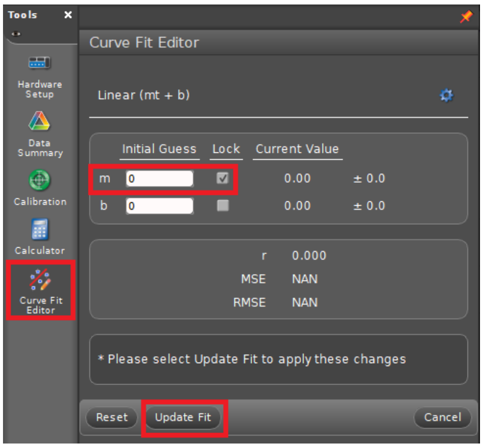
(f) Increase the frequency applied to the String Vibrator and find the second lowest frequency to maximize the amplitude.



- Skip the frequency when the end of the String Vibrator operates as an antinode.

(g) Initialize a graph of the horizontal and vertical axes with t and $F(t)$, respectively.

(h) Make linear fits with a fixed slope $m = 0$ to the $F(t)$ data to figure out the tension of the string.



(i) Replace the tension of the string and find the second lowest frequency to maximize the amplitude.

(j) Repeat the step (g)-(h).

(k) **The harmonic number n (only once during the A8)** : find the third and fourth lowest frequency to maximize the amplitude without replacing the tension.

3. **E3**

(a) Connect a String Vibrator to OUTPUT 1 of the interface.

(b) Apply the AC to the String Vibrator by Signal Generator.

(c) Increase the frequency applied to the String Vibrator and find the second lowest frequency to maximize the amplitude.

- Skip the frequency when the end of the String Vibrator operates as an antinode.

(d) Replace the length of the string and find the second lowest frequency to maximize the amplitude.

(e) Repeat the step (d).

(f) **The harmonic number n (only once during the A8)** : find the third and fourth lowest frequency to maximize the amplitude without replacing the length of the string.

References

[1] **X21.1:** *What is wave?*

[2] **X21.7:** *Definition of the Wave*

[3] **X22.1:** *Interference of Waves*

[4] **X22.2:** *Standing Waves*

[5] **X22.5:** *Resonance*

